On the magic of transforming $\bar{\rm K}$ nuclear broad quasibound states into narrow intrinsic decaying states

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(Dated: September 2, 2008)

Comments are made on Akaishi, Myint and Yamazaki's interpretation of $\bar{K}$ quasibound nuclear states as generalized Kapur-Peierls decaying states [arXiv:0805.4382]. We argue that these 'intrinsic decaying states' have little to do with low-energy $\bar{K}N$ dynamics.

PACS numbers: 13.75.Jz, 21.65.Jk, 21.85.+d

Keywords: $\bar{K}$-nuclear quasibound states

I. INTRODUCTION

Akaishi, Myint and Yamazaki (AMY) have argued recently [1], using examples taken from the phenomenology of $\bar{K}N$ and $\bar{K}NN$ systems, that quasibound states should not be defined by S-matrix poles on the appropriate Riemann sheet within nonrelativistic coupled-channel potential models. Their working example is a 2×2 coupled channel problem, $\bar{K}N - \pi\Sigma$, in which the potential in the upper $\bar{K}N$ channel is sufficiently strong to generate on its own a bound state that becomes quasibound when the two channels are coupled to each other. [This is a carricature of the $\Lambda(1405)$ $\bar{K}N$ quasibound state which is identified experimentally by observing a $\pi\Sigma$ resonance shape in various reactions.] Increasing the attraction in the upper channel, AMY observed that the $\Lambda(1405)$ pole moved to lower energies towards the $\pi\Sigma$ threshold, while becoming substantially broader. This large width of the quasibound pole state is incompatible, according to AMY, with the expected narrowness of the $\bar{K}N$ spectral shape near the $\pi\Sigma$ threshold. Instead, they suggested that Kapur-Peierls inspired Intrinsic Decaying States (IDS) replace S-matrix pole states whenever the width of the latter exceeds some relatively small value.

In this note we argue that these IDS do not emerge from any proper multichannel dynamics for the $\bar{K}N$ system at low energies, and thus IDS are not the correct theoretical construct to use for quasibound states. We also show, using a *realistic* example from low-energy $\bar{K}N$ phenomenology, that the lower among the two S-matrix poles that arise naturally in chirally motivated models becomes gradulaly narrow when the strength of the $\bar{K}N$ interaction is beefed up, joining smoothly a bound state pole below the $\pi\Sigma$ threshold. Therefore, chirally motivated models do not exhibit the pole structure that AMY were bothered by.

Although the motivation of AMY was apparently to discredit the relatively large widths of order 100 MeV found for a K^-pp quasibound state in coupled-channel three-body Faddeev calculations by Shevchenko et al. [2, 3], compared

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to the smaller width about 60 MeV found in the single-channel three-body non-Faddeev calculation by Yamazaki and Akaishi [4], we chose not to enter into argument on this point. The AMY paper does not report any new calculation for the K^-pp system beyond handwaving in terms of IDS, a concept that is refuted in the present note. For this reason we decided not to overdo our criticism of their work.

II. KAPUR-PEIERLS VERSUS GAMOW STATES

Here we sketch schematically the definitions and properties of Gamow states and of Kapur-Peierls states, without specifying the coupled channels involved in the physics of the problem. Suffice to state that our considerations hold for an effective one-channel Hamiltonian $\mathcal{H}$ which is energy dependent and is not necessarily hermitian.

A. Gamow States

Gamow states, corresponding to poles of the S matrix, were introduced by Gamow in 1928 to explain decay phenomena such as α decay of radioactive nuclei [5]. Gamow states provide a straightforward generalization of (normalizable) bound states at real energies to unstable (or quasi-) bound states and to resonances at complex energies in terms of intrinsic properties of the system and its Hamiltonian $\mathcal{H}$. Denoting by $\mid \mathcal{E}_{\rm G} >$ a Gamow state at a complex energy $\mathcal{E}_{\rm G} = E_R - {\rm i}\Gamma_R/2$, it satisfies $\mathcal{H} \mid \mathcal{E}_{\rm G} >= \mathcal{E}_{\rm G} \mid \mathcal{E}_{\rm G} >$, with a purely outgoing-wave boundary condition $< r \mid \mathcal{E}_{\rm G} > \propto \exp({\rm i}\sqrt{(2m/\hbar^2)\mathcal{E}_{\rm G}}r)$ for $r \to \infty$. The time evolution of $\mid \mathcal{E}_{\rm G} >$ is given by

$$\exp(-{\rm i}\mathcal{H}t/\hbar) \mid \mathcal{E}_{\rm G} >= \exp(-\Gamma_R t/(2\hbar)) \times \exp(-{\rm i}E_R t/\hbar) \mid \mathcal{E}_{\rm G} >, \tag{1}$$

providing an exponential decay law with a lifetime $\tau_R = \hbar/\Gamma_R$. A unique property of Gamow states is that the transition amplitude from a Gamow resonant state at a complex energy $\mathcal{E}_{\rm G}$ to a scattering state of real energy $E > 0$ is given by a Breit-Wigner (BW) amplitude

$$f_{\rm BW}(E) \;\propto\; \frac{1}{E - \mathcal{E}_{\rm G}}, \tag{2}$$

resulting in a BW resonance form of the cross section

$$\sigma_{\rm BW}(E) \;\propto\; \frac{1}{(E-E_R)^2 + (\Gamma_R/2)^2}. \tag{3}$$

Eq. (3) holds only in the immediate neighborhood of the Gamow resonance pole, so its validity is limited to *narrow* resonances, and away from thresholds. To summarize, quoting from a recent reference on the properties of Gamow states [6]: “Gamow states unify the concepts of resonance and decaying particle, and they provide a ‘particle status’ for these concepts”.

B. Kapur-Peierls States

Kapur-Peierls (KP) states were introduced in 1938 [7] as eigenstates of the Hamiltonian $\mathcal{H}$ that are regular at $r = 0$ and satisfy a purely outgoing-wave boundary condition, with a *real* wave number corresponding to a *given* real incoming energy $E_{\rm KP}$, at a radial distance r_0 outside the range of the potential. The eigenenergies $\mathcal{E}_{\rm KP}$ are complex,

depending parametrically on the real $E_{\rm KP}$. Different choices of $E_{\rm KP}$ lead to different sets of eigenenergies $\{\mathcal{E}_{\rm KP}\}$. None of these eigenenergies coincide with the pole energies of the S matrix and, thus, $\mathcal{E}_{\rm KP}$ are not related to a BW amplitude of the form Eq. (2):

$$f_{\rm BW}(E) \;\neq\; f_{\rm KP}(E) \propto \frac{1}{E-\mathcal{E}_{\rm KP}}. \tag{4}$$

We emphasize that the physical BW amplitude does not depend on the choice of r_0 and that its complex pole energy $\mathcal{E}$ does not depend on the incoming energy $E_{\rm KP}$. It is worth noting that Peierls' subsequent contributions to the subject of resonances hinged exclusively on Gamow states [8, 9]. In a posthumous publication [10], Peierls made a comment that KP resonances "are somewhat artificial because they are defined with the boundary condition that is correct only at one energy."

III. INTRINSIC DECAYING STATES

In Eq. (4), $f_{\rm KP}(E)$ is determined by a KP eigenenergy $\mathcal{E}_{\rm KP}$ that depends implicitly on the input incoming real energy $E_{\rm KP}$. AMY sought to overcome this difficulty by replacing the energy argument E by the real energy $E_{\rm KP}$ which serves in the outgoing-wave boundary condition to solve for $\mathcal{E}_{\rm KP}$. Schematically, this prescription is expressed as

$$f_{\rm AMY}(E_{\rm KP}) \propto \frac{1}{E_{\rm KP}-\mathcal{E}_{\rm KP}(E_{\rm KP})}. \tag{5}$$

It is reasonable to assume that the dominant contribution to this amplitude arises from the vicinity of the real energy $E_{\rm KP}$ that satisfies

$$\Re\,\mathcal{E}_{\rm KP}(E_{\rm KP}) \;=\; E_{\rm KP}. \tag{6}$$

Eq. (6) essentially is how AMY defined the IDS, with a complex energy $\mathcal{E}_{\rm IDS}=\mathcal{E}_{\rm KP}(E_{\rm KP})$ for the solution $E_{\rm KP}$ of Eq. (6). A more rigorous definition in terms of the Hamiltonian $\mathcal{H}$ is given below.

The case for IDS is demonstrated in Fig. 1, taken from AMY's paper [1]. Shown on the left-hand side are the trajectories of a Gamow resonance pole and of an IDS that approximately coincide at the nominal complex energy of the $\Lambda(1405)$, for a standard choice of $\bar{K}N-\pi\Sigma$ coupled-channel Yamaguchi separable interactions. These states, for this choice, are located in the fourth quadrant of the complex energy plane corresponding to the $[+,-]$ sheet, where the signs are those of $\Im k_{\bar{K}N}$ and $\Im k_{\pi\Sigma}$, respectively. The relevant portion of this quadrant is bounded from above by the real energy axis from the $\pi\Sigma$ threshold up to the $\bar{K}N$ threshold, about 100 MeV higher. The Gamow state appears as a quasibound state in the $\bar{K}N$ channel and as a BW resonance in the $\pi\Sigma$ channel. When the $\bar{K}N$ interaction strength is scaled up by a multiplicative factor f (marked along the curves in the figure) from the value $f=1.0$ it assumes for the $\Lambda(1405)$, this Gamow pole moves away from the real energy axis and its width increases. In contrast, the width of the IDS hardly increases upon applying the scaling factor f and ultimately it goes down to zero at the $\pi\Sigma$ threshold, joining there smoothly with a bound state pole (bound with respect to both thresholds). However, as shown below, this is not a consequence of the coupled-channel dynamics. It only reflects, as one approaches the $\pi\Sigma$ threshold, the weakening of the imaginary part of the effective single-channel $\bar{K}N$ Hamiltonian used to determine $\mathcal{E}_{\rm IDS}$. It accounts for phase space, not for the dynamics.

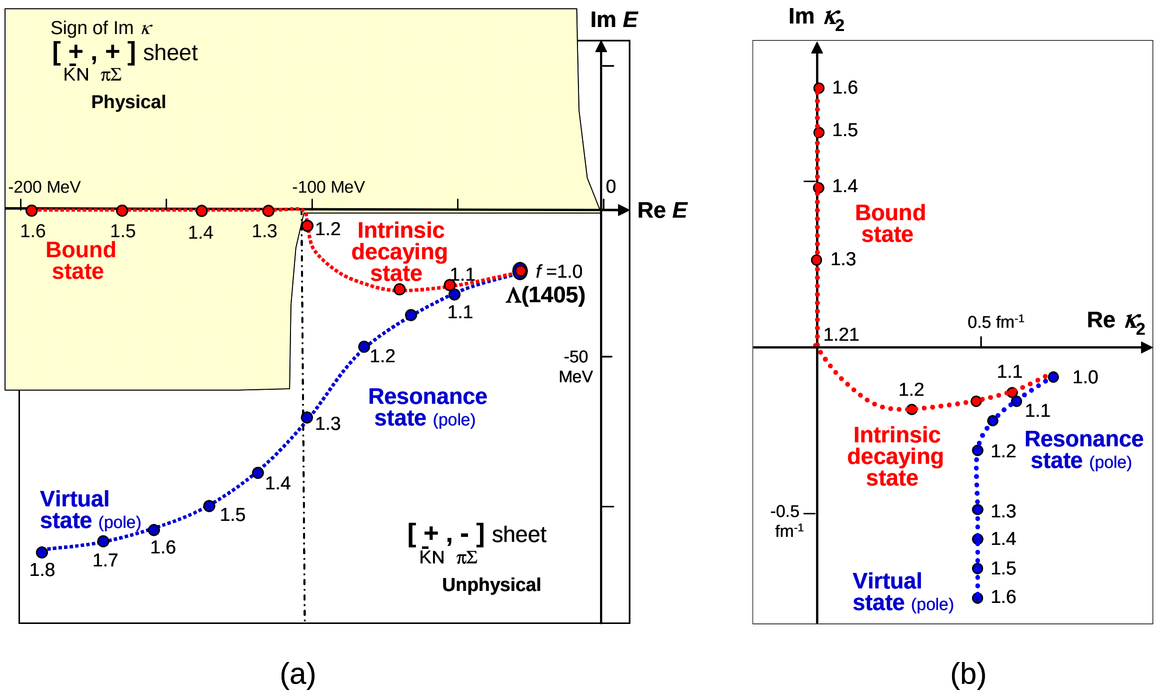

FIG. 1: Trajectories of Gamow resonance poles and of IDS: (a) in the complex energy plane, and (b) in the $k_{\pi\Sigma}$ (here denoted k_2) plane, when the strength of the $\bar{K}N$ interaction is multiplied by a factor f marking the points that form the trajectories. Figure taken from Ref. [1]. We thank Professor Yamazaki for permission to reproduce the figure.

In the $\bar{K}N - \pi\Sigma$ coupled-channel framework discussed by AMY, the Schrödinger equation is written, using Feshbach's projection operators P and Q, as

$$PHP\ \Psi_P + PVQ\ \Psi_Q = E\ \Psi_P, \quad QHQ\ \Psi_Q + QVP\ \Psi_P = E\ \Psi_Q, \tag{7}$$

where $H = T + V$ is the coupled-channel Hamiltonian. Projecting out the Q space, one obtains an effective, energy dependent Hamiltonian H_P for Ψ_P (identified with the $\bar{K}N$ wavefunction):

$$H_P(E) = PHP + PVQ\frac{1}{E - QHQ + i\epsilon}QVP, \tag{8}$$

and similarly an effective, energy dependent H_Q for Ψ_Q (identified with the $\pi\Sigma$ wavefunction):

$$H_Q(E) = QHQ + QVP\frac{1}{E - PHP + i\epsilon}PVQ. \tag{9}$$

Note that these effective Hamiltonians are not necessarily hermitian. The Gamow resonance states in the $[+,-]$ sheet of the complex energy plane in Fig. 1 are eigenstates of the coupled channel Hamiltonian system Eq. (7) and are also eigenstates of each one of the channel Hamiltonians:

$$H_P(\mathcal{E}_{\rm G})\Psi_P = \mathcal{E}_{\rm G}\Psi_P, \quad H_Q(\mathcal{E}_{\rm G})\Psi_Q = \mathcal{E}_{\rm G}\Psi_Q, \tag{10}$$

with a common eigenenergy $\mathcal{E}_{\rm G}$ and with Ψ_P and Ψ_Q satisfying each an outgoing-wave boundary condition.

The operational definition of IDS, Eq. (6), was done in terms of a projection onto the P channel only. It is easily shown to be equivalent to the following, general definition:

$$H_P(\Re\mathcal{E}_{\rm IDS})\Psi_P = \mathcal{E}_{\rm IDS}\Psi_P. \tag{11}$$

In order to retain a meaning in a coupled-channel formulation, an extension onto the Q space is required, which AMY hardly discussed. A sensible extension in the spirit of the underlying KP philosophy is to require

$$H_Q(\Re\mathcal{E}_{\rm IDS})\Psi_Q = \mathcal{E}_{\rm IDS}\Psi_Q. \tag{12}$$

However, since $\Re\mathcal{E}_{\rm IDS}$ is sought between the two thresholds, the effective Hamiltonian in Eq. (12) is *real* and for the Yamaguchi-type separable potentials used by AMY the resultant eigenenergy $\mathcal{E}_{\rm IDS}$ (if any) is also real, differing from the complex value satisfying Eq. (11). This argument demonstrates that the IDS concept cannot be extended satisfactorily from one channel to include all the relevant channels in the coupled-channel dynamics. IDS, therefore, are not a property of the coupled-channel Hamiltonian, nor of the S matrix that determines the physical spectral shapes and cross sections.

IV. CHIRALLY MOTIVATED MODELS

Modern chirally motivated coupled-channel models give rise to *two* Gamow states that dominate low-energy $\bar{K}N$ dynamics. For a recent review, see Ref. [11]. One state corresponding to an $I=0$ $\bar{K}N$ quasibound state is generated dynamically from the strongly attractive interaction in the $\bar{K}N$ channel. However, this state cannot be identified with the $\Lambda(1405)$. The other Gamow state, on the same Riemann sheet, originates from a resonance in the $\pi\Sigma$ channel and it corresponds to the physical $\Lambda(1405)$. Here we would like to follow the movement of these two poles in the complex energy plane upon changing the strength of the interaction in a chirally motivated coupled-channel model developed recently by one of us [12].

The model consists of 10 coupled channels, made out of the two-body systems $\bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Lambda, \eta\Sigma, K\Xi$ with zero total charge. It fits well all the low-energy K^-p scattering and reaction data except for the perennially irreproducible $1s$ atomic width, and it reproduces reasonably well the $\pi\Sigma$ spectrum shape which is identified with the $\Lambda(1405)$ resonance. To be specific, we use the parameter set that gives $\sigma_{\pi N} = 40$ MeV (see Table 2 in Ref. [12]). Other parameter sets that fit the data equally well produce similar trends to that discussed below. The model yields two Gamow poles in the fourth quadrant of the $[+,-]$ Riemann sheet, with composition dominated by $\pi\Sigma$ and $\bar{K}N$ channels. These pole positions, at $(E_R, -{\rm i}\Gamma_R/2) = (1391, -{\rm i}51)$, $(1450, -{\rm i}69)$ (in MeV), respectively, are shown in Fig. 2 together with the trajectories followed by these poles upon multiplying the $\bar{K}N$ ($\pi\Sigma$) interactions by a scaling factor as indicated in the upper (lower) part. The $\Lambda(1405)$ resonance corresponds to the lower pole at $(1391, -{\rm i}51)$ MeV. The upper pole appears in this model above the $\bar{K}N$ threshold, as it does in other models (see Fig. 8 in Ref. [11] for a compilation of results from various chiral models), and it is more likely to be associated with $\bar{K}$ quasibound states in nuclei. Fig. 2 shows that this upper pole reacts differently to scaling of various pieces of the interactions: increasing the strength of the $\bar{K}N$ interactions, it drifts to lower energies below the $\bar{K}N$ threshold while becoming very broad, whereas increasing the strength of the $\pi\Sigma$ interactions, it remains above the $\bar{K}N$ threshold and its width becomes vanishingly small. In contrast, the lower pole drifts to lower energies, approaching the real axis, and for a sufficiently strong coupling it forms a bound state below the $\pi\Sigma$ threshold. A similar dependence of pole positions on scaling factors was observed in a recent study of Gamow states within a simplified model used to test phenomenological methods of extracting resonance parameters in meson-nucleon reactions [13]. While this study presents the behavior of resonance poles in a more general context, here we employed a realistic multichannel model that is based on chiral

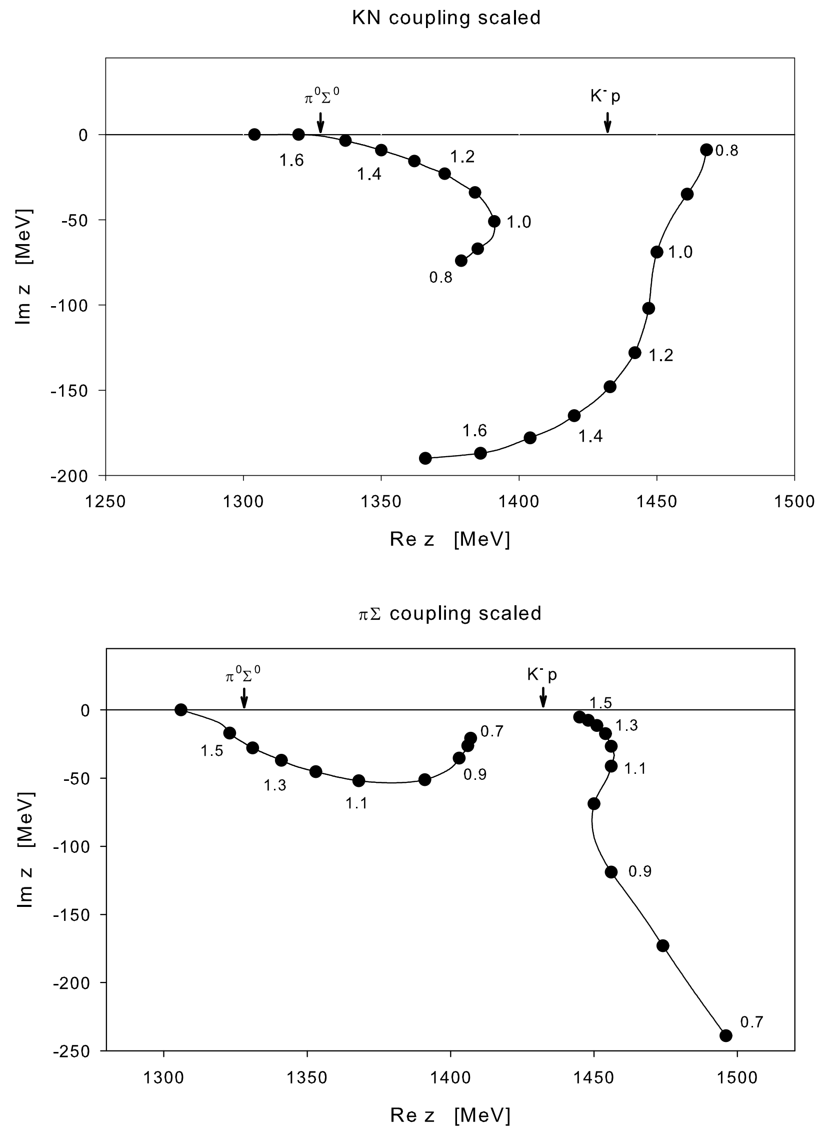

FIG. 2: Trajectories of Gamow poles in the complex energy (z) plane on the Riemann sheet $[\Im k_{\bar{K}N}, \Im k_{\pi\Sigma}] = [+,-]$ using the 10-channel model of Ref. [12]. The upper (lower) part depicts the motion upon scaling the $\bar{K}N$ $(\pi\Sigma)$ interaction strengths by a multiplicative factor marking the points that form the trajectories. The $\pi^0\Sigma^0$ and K^-p thresholds are marked by arrows.

dynamics. We conclude that Gamow states associated with low-energy $\bar{K}N$ phenomenology display a more subtle pattern than the classification made by AMY.

V. CONCLUDING REMARKS

In this brief note we argued against the applicability of the IDS concept introduced by AMY to describe low-energy $\bar{K}N$ dynamics. Gamow resonance states and Gamow quasibound states, associated with Gamow poles, are the only quantum states that provide a proper generalization of normalizable bound states. Gamow states are nonrenormalizable eigenstates of the multichannel Hamiltonian $\mathcal{H}$ and satisfy eigenstate equations with outgoing-wave boundary conditions, Eqs. (10), in each channel. Gamow states are independent of any reaction mechanism

by which one seeks to establish such resonances or quasibound states. To fit and interpret production or formation reaction cross sections in terms of quantum states that are intrinsic property of $\mathcal{H}$, obviously one needs to superimpose the constraints of phase space which are specific to that given reaction. It is wrong, however, to incorporate phase space constraints imposed by the reaction which generates such quantum states into their definition.

IDS are *not* eigenstates of $\mathcal{H}$ in *all* the relevant channels. AMY conjecture that channel P provides the doorway for forming a dynamical entity in the reaction they chose to analyze, and that's why they geared the IDS to satisfy an eigenstate equation, Eq. (11), in channel P. Suppose that we conjecture that channel Q provides the doorway for forming the same dynamical entity in a different reaction; are we then justified to define IDS by satisfying an eigenstate equation in channel Q? If we do, the two IDS will be different from each other and would not qualify to describe the coupled channel dynamics. The example in Sect. IV of two dynamical poles, one arising from P-channel interactions while the other one from Q-channel interactions, provides some justification to exploring more than one production/formation mechanism in the study of $\bar{K}$ nuclear quasibound states. For these quasibound states to reflect the coupled-channel Hamiltonian dynamics, they must arise with the same eigenenergy in all channel spaces, something that IDS are unable to deliver in few-body systems where coupled-channel dynamics plays a crucial role.

Acknowledgments

This work was supported in part by the GA AVCR grant IAA100480617 and by the Israel Science Foundation grant 757/05.

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